

Magnetic Susceptibility in Strongly Coupled Systems

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We study the magnetic susceptibility at large 't Hooft coupling by computing the correlation function of the magnetizations in the strongly coupled Maxwell theory in large-N limit with finite temperature and chemical potential, within the framework of the AdS/CFT correspondence. We show that in strong coupling limit the magnetic susceptibility is independent to the temperature and be universal, measured in the unit of magnetic permeability of the bulk space. A comparison with the weak coupling system, the Pauli paramagnetic susceptibility, is also discussed.

I. INTRODUCTION

Magnetic susceptibility χ is an important property of materials indicating the magnetic response to an applied external magnetic field. In microscopic picture, χ measures the propagation of the collective wave mode of local magnetization \mathbf{M} , called spin wave. Its quanta are the magnons, up to a gyromagnetic ratio constant, the magnetic susceptibility and the spin-spin correlation function can be viewed as one thing. In fact, the magnetic susceptibility is the magnetization or spin transport. Like other theoretical studies of transport coefficients, e.g. see [1, 2], the magnetic susceptibility could be calculated by using the standard perturbative technique in quantum field theory, which is based on the precondition that the magnetizations or, equivalently, the spins taken by constituent fermions interact weakly with each other. The condition is fulfilled in the Fermi-liquid theory, which is a theory in the vicinity of a trivial fixed point [3]. When the interaction becomes stronger, the calculations are notoriously difficult. However, it is conjectured that another non-trivial fixed point exists [4] and corresponds to a strongly coupled conformal fields theory that is dual to a string/M theory in an AdS space, the AdS/CFT correspondence [5–7]. These two fundamentally different fixed points correspond to the extremely weak and strong coupling limit [3].

An example of the study of the magnetic susceptibility in extremely weak coupling limit is the Pauli paramagnetism, which describes χ in electron gas [8]. The validity of the weakly interacting description is based on the fact that the Coulomb interactions are effectively screened, so the Coulomb interaction becomes a short range force characterized by the Debye mass. But it is known that the magnetic interactions which are mediated by the magnons can not be effectively screened and spoil the normal Fermi-liquid behavior of the system [9], therefore, the non-perturbative effects in magnetic susceptibility are thought to be important, especially in strongly coupled system. In the extremely strong coupling limit, the 't Hooft coupling tends to infinity, the string/M theory is reduced to a classical (super)gravity, so it allows us to do calculation of the correlation functions in the limit.

One of the famous predictions [10, 11] of the AdS/CFT correspondence was that the ratio of the shear viscosity η to the entropy density s , in $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory at large 't Hooft coupling with finite temperature, equals to $1/4\pi$ (in natural units), it is a universal quantity independent with the microscopic details, which agrees well with the observation from the strongly coupled quark-gluon-plasma produced in relativistic heavy ion collision [12]. The strongly coupled QCD theory is an important area for applying the results. The surprising success of the AdS/CFT correspondence at low energies is probably the result of the universality in its predictions. Note that the magnetic susceptibility is dimensionless, so a naive guess is that it may be universal as well in the prediction from the approach, it is very interesting to check this idea by detail calculations. Another motivation for doing the calculation is that the topic of magnetic aspects of the quark matter in the phase diagram of QCD have attracted many interests, e.g. see [13–18].

In this paper, we will work in the framework of AdS/CFT correspondence at large 't Hooft coupling limit with finite temperature and chemical potential [19]. The idea to calculate the magnetic susceptibility in this framework is simple. Similarly to the standard procedure in calculating the other transport coefficients in Minkowski prescription [20], one places the magnetization M_i on the 4-dimensional boundary that couples to the magnetic field H_i which propagates in the 5-dimension bulk AdS space. One can write down the action of the magnetic fields in the bulk space deduced from the Maxwell action in the AdS background. Depending on the thermodynamical variables of the system, we need to place a Schwarzschild or charged Reissner-Nordstrom black hole into the AdS space, which corresponds to introduce finite temperature and/or chemical potential, respectively. The two point correlation function (in Minkowski space) of M_i can be computed by performing the functional derivative with respect to the magnetic field H_i as a source on the boundary. Here the magnetic fields H_i in the bulk are dual to M_i , which is analogous to the case that we compute the correlation function of charged currents J_i where the gauge fields A_i in the bulk are dual to J_i .

The paper is organized as follows. In section II we briefly define the magnetic susceptibility from the linear response theory and review the computing framework of the Green's functions from AdS/CFT correspondence in Minkowski prescription. In section III, we perform a detail calculation to the magnetic susceptibility in two cases, the system with

temperature and temperature together with chemical potential. We also compare our result with the one computed from the weakly coupled limit, the Pauli paramagnetic susceptibility. Section IV contains the conclusions.

II. PRELIMINARIES

A. Magnetic susceptibility in the linear response theory

In this section, we set up a field theoretical framework for the response of a system at equilibrium to small perturbations. The framework allows us to relate a two point correlation function of magnetizations to the magnetic susceptibility of the system.

Let us consider the response of the system to the presence of a weak external magnetic field $H^i(x)$ which couples to a the magnetization M_i . Then the Hamiltonian is perturbed by a term

$$\delta\mathcal{H} = \int d^4x M_i(x) H^i(x), \quad (1)$$

where the index $i = 1, 2, 3$. The standard perturbation theory in textbook of quantum mechanics tells us that it produces a change in the expectation value of the operators

$$\delta\langle M_i(x) \rangle = \int d^4x' \tilde{G}_{ij}^R(x, x') H^j(x') + \mathcal{O}(H^2), \quad (2)$$

in which

$$\tilde{G}_{ij}^R(x, x') = -i\theta(t - t') \langle [M_i(x), M_j(x')] \rangle \quad (3)$$

is the retarded Green's function. The result can also be found by using the Kubo formula, which tells us that to first order in the time-dependent perturbation, the induced vector current (here it is the perturbative wave of magnetization $M_i(x)$, or the spin wave current) is equal to retarded correlator to the vector current with the perturbation evaluated in equilibrium. The Fourier transformed linear response then takes a simple form

$$\delta M_i(k) = G_{ij}^R(k, 0) H_j(k) + \mathcal{O}(H^2), \quad (4)$$

where the Fourier transformation of the retarded Green's function is

$$G_{ij}^R(k) = \int d^4x e^{-ikx} \tilde{G}_{ij}^R(x, 0). \quad (5)$$

To see the relation between the retarded Green's function to the magnetic susceptibility χ_{ij} , we write down its definition

$$M_i(k) \equiv \chi_{ij}(k) H_j(k), \quad (6)$$

which χ_{ij} is a second rank tensor. Compared with Eq.(4), consider that here the external perturbation is weak, at linear level, the magnetic susceptibility tensor is identified with the retarded Green's function, i.e. the two point magnetization-magnetization correlation function

$$\chi_{ij}(k) = G_{ij}^R(k, 0). \quad (7)$$

B. Minkowski correlators in AdS/CFT correspondence

In order to calculate the two point magnetization-magnetization correlation function of a thermal strongly coupled system in Minkowski space, one need to discuss in detail a prescription for computing a two-point Green's function from gravity, followed by the AdS/CFT correspondence. One can write the AdS/CFT correspondence as the equality in Euclidean version

$$\langle e^{\int_{\partial\mathcal{M}} M_i H_0^i} \rangle = e^{-S_{cl}[H]}. \quad (8)$$

The left hand side is a generating functional for the correlators of magnetization in the boundary field theory, which is conjectured as a $\mathcal{N} = 4$ SU(N) SYM theory at large N limit. When the 't Hooft coupling $g_{YM}^2 N$ tends to infinity, the right hand side tends to the action of the classical Einstein (super)gravity, and the external magnetic field H propagates in the bulk AdS_{d+1} space, with its boundary condition H_0 couples to the magnetization M_i on the boundary $\partial\mathcal{M}$ of the AdS space. In order to introduce a finite temperature to the system, one has to place a black hole to the AdS space, the metric in Minkowski version can be written as

$$ds^2 = \frac{(\pi T R)^2}{u} (-f(u)dt^2 + dx^2 + dy^2 + dz^2) + \frac{R^2}{4u^2 f(u)} du^2, \quad (9)$$

where for Schwarzschild-AdS background we have $f(z) = 1 - u^2$ and $u = r_0^2/r^2$, r_0 is the radius of the horizon of the black hole, in which $T = r_0/\pi R^2$ is the Hawking temperature, the horizon locates at $u = 1$, the boundary at $u = 0$.

As proposed by Son and Starinets [20], to generalize the AdS/CFT correspondence from the Euclidean to Minkowski version, formally we have the relation

$$\langle e^{i \int_{\partial\mathcal{M}} M_i H_0^i} \rangle = e^{i S_{cl}[H]}, \quad (10)$$

together with the incoming-wave boundary condition at the horizon, i.e. all modes are absorbed into the black hole horizon but no ones can emit. By using the Eq.(7) and Eq.(10), the retarded Green's function, and then the magnetic susceptibility in a strongly coupled system can be computed from the second functional derivative of S_{cl} with respect to the boundary value H_0 ,

$$\chi_{ij} = -2 \frac{\delta^2 S_{cl}[H]}{\delta H_0^i \delta H_0^j} \Big|_{u \rightarrow 0}. \quad (11)$$

III. HOLOGRAPHIC CALCULATION

A. Finite Temperature

In this section, we work on the 5-dimensional Schwarzschild-AdS background and consider the perturbations of magnetic field H^i in it. Our starting point is the 5-dimensional Maxwell action in the background Eq.(9),

$$S = -\frac{1}{4g_{YM}^2} \int d^5x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}, \quad (12)$$

where

$$g_{YM}^2 = 16\pi^2 R/N^2 \quad (13)$$

is the coupling constant. In this paper, what we are interested in is the correlator of magnetizations coupled to the magnetic fields which are directly observed physical quantities unlike the gauge potential A_μ , so we will use the electric and magnetic fields (E_i, H_i) as fundamental dynamical variables. One can rewrite the action as

$$S = -\frac{1}{2g_{YM}^2} \int d^5x \sqrt{-g} (\epsilon_0 E_i E^i - \mu_0 H_i H^i), \quad (14)$$

where ϵ_0 is the electric permittivity, μ_0 the magnetic permeability of the vacuum in the bulk space. Here we assume that the backreaction of the source on the boundary to the bulk electromagnetic fields is small, the electric and magnetic wave that will propagate in the bulk along u axis are almost purely transverse, so we shall set $E_u = H_u = 0$, the physical independent fields are those with index $i = x, y, z = 1, 2, 3$. The physical components are defined as

$$\sqrt{\epsilon_0} E^i = F^{i0}, \quad \sqrt{\mu_0} H^i = \frac{1}{2} \epsilon^{ijk} F_{jk},$$

where $\epsilon^{ijk} = 1$ for that the order of indices (ijk) are an even/odd permutation of (123) . One can use the Fourier decomposition

$$P_i(x, u) = \int \frac{d^4 K}{(2\pi)^4} e^{-i\omega t + i\mathbf{k} \cdot \mathbf{x}} P_i(K, u), \quad P = H \text{ or } E. \quad (15)$$

By locally using the Maxwell equation in 4-dimensional space

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad (16)$$

to replace the transverse electric fields E_i with magnetic fields H_i locally. Then the action can be written as

$$S = -\frac{1}{2g_{YM}^2} \mu_0 \int du \int \frac{d^4 K}{(2\pi)^4} \sqrt{-g} (\epsilon_0 \mu_0 \omega^2 - k^2) \frac{1}{k^2} H_i(K, u) H^i(K, u). \quad (17)$$

Without loss of generality, one can set the speed of light $c^2 = (\epsilon_0 \mu_0)^{-1} = 1$ in the bulk space, so we have

$$S = \frac{1}{2g_{YM}^2} \mu_0 \int du \int \frac{d^4 K}{(2\pi)^4} \sqrt{-g} \frac{K^2}{k^2} H_i(K, u) H^i(K, u), \quad (18)$$

where $K^2 = -\omega^2 + k^2$, we denote $K_\mu = (\omega, \mathbf{k})$ locally as a 4-momentum. The magnetic fields can be decomposed as

$$H^i(K, u) = h_K^i(u) H_0^i(K), \quad (19)$$

note that h_K^i equals to 1 at the boundary $u = 0$,

$$\lim_{u \rightarrow 0} h_K^i(u) = 1. \quad (20)$$

The equations of motion of magnetic fields $H_i(K, u)$ in the extra dimension u are given by the decoupled equations of motion of $h_K^i(u)$

$$\frac{1}{\sqrt{-g}} \partial_u (\sqrt{-g} g^{uu} \partial_u h_K^i) - g^{\mu\nu} K_\mu K_\nu h_K^i = 0. \quad (21)$$

Introducing dimensionless energy and momentum in unit of temperature

$$\omega = \frac{\omega}{2\pi T}, \quad \mathbf{k}_i = \frac{\mathbf{k}_i}{2\pi T}, \quad (22)$$

substituting the metric Eq.(9) into Eq.(21), we have

$$(h_K^i)'' + \left(\frac{f'}{f} - \frac{1}{u} \right) (h_K^i)' + \left(\frac{\omega^2}{uf^2} - \frac{\mathbf{k}^2}{uf} \right) h_K^i = 0, \quad (23)$$

in which the prime stands for the derivative with respect to u . The Eq.(23) is a second-order differential equation for $h_K^i(u)$ in which at the horizon $u = 1$ is a singular point, and behaves as $h_K^i = (1-u)^\nu F^i(u)$, where $F^i(u)$ is a regular function. There are only two values of $\nu_\pm = \pm i\omega/2$, and the incoming wave boundary condition at the horizon is ν_- . Then we obtain the equation for $F^i(u)$,

$$F'' + \left(-\frac{1+u^2}{uf} + \frac{i\omega}{1-u} \right) F' + \left(-\frac{i\omega}{2uf} \right) F + \frac{\omega^2 [4 - u(1+u)^2]}{4uf^2} F - \frac{\mathbf{k}^2}{uf} F = 0, \quad (24)$$

Since the three equations are decoupled and identical, we have omitted the superscript i and denoted the solution as F . In the low frequency and long wavelength limit, the ω and \mathbf{k} can be considered small, we solve the equation perturbatively by expanding the solution F in powers of these small parameters

$$F(u) = F_0 + \omega F_1 + \mathbf{k}^2 G_1 + \omega^2 F_2 + \omega \mathbf{k}^2 H_1 + \dots \quad (25)$$

The leading order contribution is given by first three functions F_0, F_1, G_1 , which can be solved explicitly, the integration constants can be fixed by requiring that these functions are regular at the horizon $u = 1$, and vanish in the limit $u \rightarrow 1$ (except F_0). We obtain

$$F_0 = C, \quad F_1 = -\frac{iC}{2} \log \frac{1+u}{2}, \quad G_1 = -C \log \frac{1+u}{2}. \quad (26)$$

The constant C is determined by the boundary condition Eq.(20), so we have

$$C = \frac{1}{1 + (\frac{i}{2}\omega + \mathbf{k}^2) \log 2}. \quad (27)$$

Near the boundary, the radial derivative of the field behaves as

$$\lim_{u \rightarrow 0} \partial_u h_K = -\mathbf{k}^2 - \frac{\omega^2}{4} \log 2 + \frac{i}{2} \omega \mathbf{k}^2 \log 2. \quad (28)$$

at leading order

$$\lim_{u \rightarrow 0} \partial_u h_K = -\mathbf{k}^2 + \dots \quad (29)$$

where ... denotes the higher order corrections, $\mathcal{O}(\omega^2)$ and $\mathcal{O}(\omega \mathbf{k}^2)$. Substituting the solution into Eq.(18) and Eq.(19) and integrate u by part, we get

$$S = \frac{1}{2g_{YM}^2} \mu_0 \int \frac{d^4 K}{(2\pi)^4} \sqrt{-g} \frac{1}{k^2} H_0^i g^{ij} \left[g^{uu} h_{-K}^i \partial_u h_K^j \right] H_0^j. \quad (30)$$

So according to the Eq.(11) and Eq.(13), we have

$$G_{ij}^R = \frac{N^2 \delta_{ij}}{32\pi^2} \mu_0 + \dots \quad (31)$$

We see that the correlation functions is isotropic, the magnetic susceptibility of the system can be written as a scalar

$$\chi = \frac{N^2}{32\pi^2} \mu_0 + \dots \quad (32)$$

B. Finite Temperature and Chemical potential

To generalize this result to a system with finite density, one need to replace the Schwarzschild black hole by a charged black hole, namely, the Reissner-Nordstrom-AdS (RN-AdS) background, which has the same form as Eq.(9) with a different structure of horizon

$$f(u) = (1-u)(1+u-au^2), \quad (33)$$

where a is parameter that relates to the charge of the black hole. The temperature and chemical potential of the system can be now written as

$$T = \frac{1}{2\pi b} \left(1 - \frac{a}{2}\right), \quad \Sigma = \frac{1}{2b} \sqrt{\frac{3a}{2}}, \quad (34)$$

in which b is another parameter related to the mass of the black hole [19]. The calculating process is similar, we need to solve the differential equations Eq.(23) by using Eq.(33). Similarly, the solution is found to be

$$h_K(u) = C(1-u)^{-i\omega/2} \left[1 + \omega F_1 + \mathbf{k}^2 G_1 + \mathcal{O}(\omega^2, \omega \mathbf{k}^2 \dots) \right], \quad (35)$$

with

$$C = \frac{1}{1 - \frac{1}{4}\omega \left[\pi + i \log(a-2) \right] - \frac{3i\omega + 4\mathbf{k}^2}{2\sqrt{-1-4a}} \left[\tan^{-1} \left(\frac{2a-1}{\sqrt{-1-4a}} \right) + \tan^{-1} \left(\frac{1}{\sqrt{-1-4a}} \right) \right]}, \quad (36)$$

$$F_1 = -\frac{i}{4} \log \left(\frac{2-a}{1+u-au^2} \right) + \frac{3i}{2\sqrt{-1-4a}} \left[\tan^{-1} \left(\frac{2au-1}{\sqrt{-1-4a}} \right) - \tan^{-1} \left(\frac{2a-1}{\sqrt{-1-4a}} \right) \right], \quad (37)$$

$$G_1 = \frac{2}{\sqrt{-1-4a}} \left[\tan^{-1} \left(\frac{2au-1}{\sqrt{-1-4a}} \right) - \tan^{-1} \left(\frac{2a-1}{\sqrt{-1-4a}} \right) \right]. \quad (38)$$

The behavior near the boundary is

$$\lim_{u \rightarrow 0} \partial_u h_K = -\mathbf{k}^2. \quad (39)$$

Differ from Eq.(29), there are no higher order corrections such as $\mathcal{O}(\omega^2)$, $\mathcal{O}(\omega \mathbf{k}^2)$. Applying the prescription formulated in the previous section, one finds

$$\chi = \frac{N^2}{32\pi^2} \mu_0, \quad (40)$$

which is our final result for the magnetic susceptibility at large 't Hooft coupling $g_{YM}^2 N \gg 1$. It can be regarded as a nontrivial prediction from the strongly coupled $\mathcal{N} = 4$ SYM theory at finite temperature and chemical potential. The first observation is that in this limit χ is independent with the temperature and the 't Hooft coupling, it is so simple and be a universal quantity. It is measured in the unit of the magnetic permeability μ_0 of the bulk space.

The result is positivity, if we have an analytic continuation for the result from large N to finite N , the system would be paramagnetic. Note that in the weak coupling limit, the quasi-particle gas is paramagnetic, the Pauli paramagnetism [8], it is interesting to compare the Eq.(40) with the Pauli paramagnetic susceptibility. In this weak coupling regime, the χ_{Pauli} comes from the contribution of free quasi-particles near the Fermi surface [8]

$$\chi_{Pauli} = \mu_0 \mu_B^2 \rho, \quad (41)$$

where μ_0 is the vacuum permeability, $\mu_B^2 = g_{YM}^2 N / 4m^2$ is the Bohr magneton, m the effective mass of the quasi-particle, and

$$\rho = -2N \int \frac{d^3k}{(2\pi)^3} \frac{\partial n_k}{\partial \omega_k} = \frac{N k_F m}{\pi^2}, \quad (42)$$

is the density of states near the Fermi surface, where k_F is the fermi momentum, N the number of species of the fermions. Finally we get

$$\chi_{Pauli} = \frac{N}{4\pi^2} \mu_0 \frac{(g_{YM}^2 N) k_F}{m}, \quad g_{YM}^2 N \ll 1. \quad (43)$$

The Eq.(40) and Eq.(43) implies that in strong coupling regime the “quasi-particle” (if we can still denote them by this name) becomes heavy so that the effective mass is comparable to the order of the numerator near the Fermi surface, i.e. $m \sim \mathcal{O}(g_{YM}^2 k_F)$, and $m = 8g_{YM}^2 k_F$ for $g_{YM}^2 N \rightarrow \infty$. Note that the life-time of quasi-particle is $\tau \sim 1/m$, so the notion of the long-lived quasi-particle at the Fermi surface does not hold any more in the strongly coupled system, instead of a broadened spectral density and/or smeared Fermi surface, which has been observed in the studies on strongly coupled non-Fermi-liquid system [21–24]. We expect that χ behaves similarly in a non-Fermi-liquid system.

IV. CONCLUSION

In this paper, we have calculated the real-time correlation function of the magnetization \mathbf{M} , i.e. the magnetic susceptibility χ , in the $\mathcal{N} = 4$ SYM theory at finite temperature and chemical potential by using the Minkowski AdS/CFT prescription. We show that in extremely strong coupling limit, the magnetic susceptibility, measured in the unit of magnetic permeability in the bulk space, does not vary with the temperature and 't Hooft coupling. It is found to be universal and independent from the microscopic details. We expect that our result can be extended and applied to the strongly coupled quark-gluon-plasma and the non-Fermi-liquid system observed in strange metal phase of cuprate superconductors and many heavy fermion materials.

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